Why is it so hard to “optimize”

- Static solutions verses dynamic
- Deterministic approach verses Stochastic
- Single level vs. multi level (multi echelon inventory systems)
- Single product or multiple products
- ...

*A theory should be as simple possible, but no simpler.*

Albert Einstein
Demonstration: The Beer Game

Try to follow this “simple” example:
Very Basic Inventory Management

**Scenario:**
Minimize the cost of a single “node” in the chain.

Assume that you provide a product to a loyal customer. You know the demand for your product and you need to decide how to stock yourself to be able to provide the product at a minimum cost to you.
Very Basic Inventory Management

- No backorders
- Real ordering costs
- Real cost of carrying inventory (o.w. use JIT)
- No limit on order size
- Constant demand rate!
- Fixed lead time!
- Single item inventory

**EOQ or POQ models:**

- Inventory level
- Time
- \( Q/D, 2Q/D, 3Q/D, 4Q/D \)
- \( p-d, t_1, t_2 \)
Very Basic Inventory Management

Static, deterministic, single level, single product.

Holding cost

Ordering cost

\[ Q^* = \sqrt{\frac{2AD}{h}} \]

\[ Y(Q) \]

\[ Q^* = 169 \]

\[ c \]

\[ A/Q \]

\[ hQ/2D \]
Somewhat Basic Inventory Management

Scenario:
Now your customer sends you his list of orders for the next few weeks in advance. Every week you need to supply that demand. Due to your cost structure you may want to order your material using a different schedule.
Somewhat Basic Inventory Management

Static, deterministic, variable demand, single level, single product.

Example:

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<th>3</th>
<th>4</th>
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</tr>
</tbody>
</table>

How realistic is this optimal solution?
Somewhat Basic Inventory Management - Additional Issues

- Quantity discount
- Variable (uncertain) lead time
- Cost of lead-time commitment
- Cost of contract flexibility (order quantities).
- Many more…
Basic Stochastic Model – still Static Newspaper Boy Model

★ Scenario:
As a “node” in the supply chain you need to be able to provide an “uncertain” demand from your customer.

The demand could come unexpectedly (but also could be scheduled in advance), and all you are trying to do is make sure that you can face the next order “effectively”.
Basic Stochastic Model – still Static Newspaper Boy Model

★ Static, Single period but stochastic!
(Newspaper boy model)

Risk of Holding

$$G(Q^*) = P\{X \leq Q^*\} \leq \frac{C_s}{C_o + C_s}$$

Risk of shortage

$$\Phi(Z)$$
Less basic Stochastic Model — still Static Base Stock Model

★ Scenario:
You are a special node in the chain – possibly a retailer. You sell high cost items (refrigerators) and need to have certain number of items in your display room or inventory.
Every time that you sell a refrigerator, you order a new one for your showroom.
Now you need to decide how many refrigerators you keep in your inventory.
Main issue – you fight against the lead time.
Less basic Stochastic Model – still Static Base Stock Model

Base Stock Model: No ordering costs
- **Multi period**!
- Order one item at a time. Prelude to a Kanban system.
- Demand is approximated by a distribution

• Manageable but hard:
  • Concerns about cost and Service Level!
Advanced Stochastic Model – still Static (Q,R) Model

🌟 Scenario:
As a node in the supply chain you need to satisfy uncertain demand using a “continuous review” approach.

You would like to protect yourself against “excessive” demand, but also control your inventory size.

Again, you are trying to beat the “lead time” issue.
Advanced Stochastic Model – still Static (Q,R) Model

- Solvable but hard.
- Produces a fixed strategy for many periods.
Advanced Stochastic Model — still Static
(Q,R) Model

- Multi period
- Service level or cost
- Considers order quantity AND reorder point.
- Generates “Safety Stock”
Advanced Stochastic Model – still Static (Q,R) Model
Dynamic Models – Multi Station Flow

⭐️ Scenario:

• You are responsible for several “nodes” in the supply chain.
• You would like to reach an overall good performance.
• You know the rates of flow of your goods (i.e. demand per time unit) and you have good control over each node.
• Now you have “many” parameters to control!
Dynamic Models – Multi Station Flow

- “horizontal” or one-way flow

- Based on rates of service
- Consider Bottleneck Rate and “Raw processing time”
Dynamic Models – Multi Station Flow

- Provide envelop of performance (one supplier “node” at a time)
Dynamic Models – Impeding Factors

Individual Node

- But, things are never perfect - variability
- Natural variability
- Unexpected delays (issues of reliability/quality)
- Planned delays (setups, batching etc. - management)
Dynamic Models – Flow between Nodes

Flow Variability:
• Arrivals variability
• Departure variability
• Utilization

\[ c_d^2(i) = u^2 c_e^2(i) + (1 - u^2) c_a^2 \]
Illustrating Flow Variability

- Low variability arrivals
  - smooth!
  - High variability arrivals
  - bursty!
Putting it all together

• Considers individual variability of all types
• Considers operational variability (Flow)
• Combining each node to the next one
• Allows quantitative modeling (no need for simulation)
Putting it all together - Example